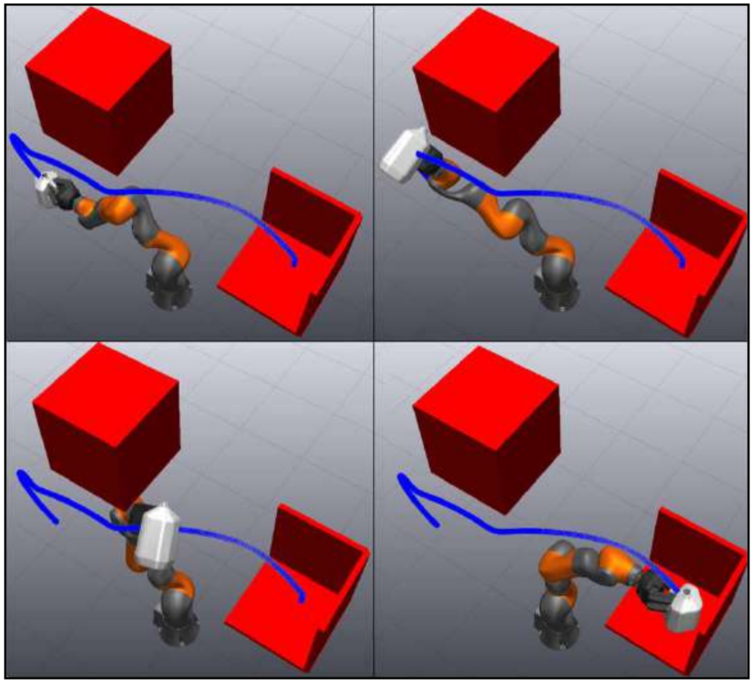
Constrained Unscented Dynamic Programming



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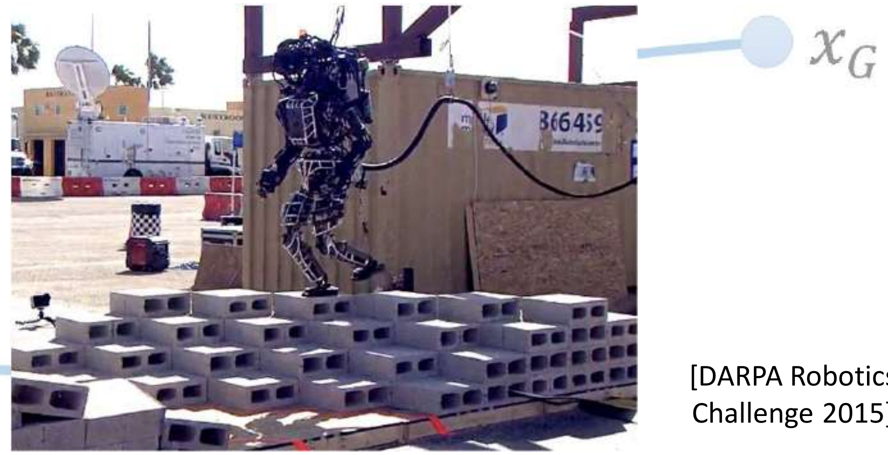
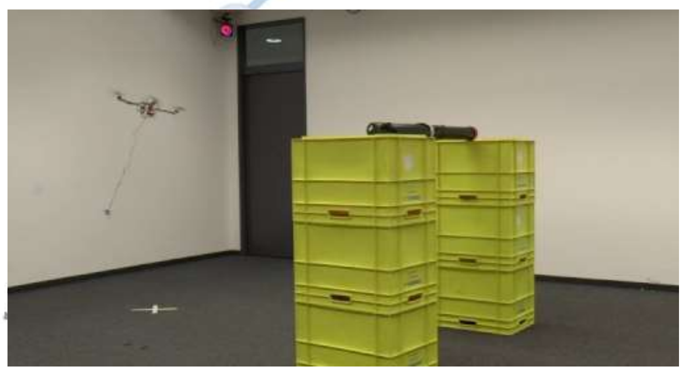
Harvard Agile Robotics Lab

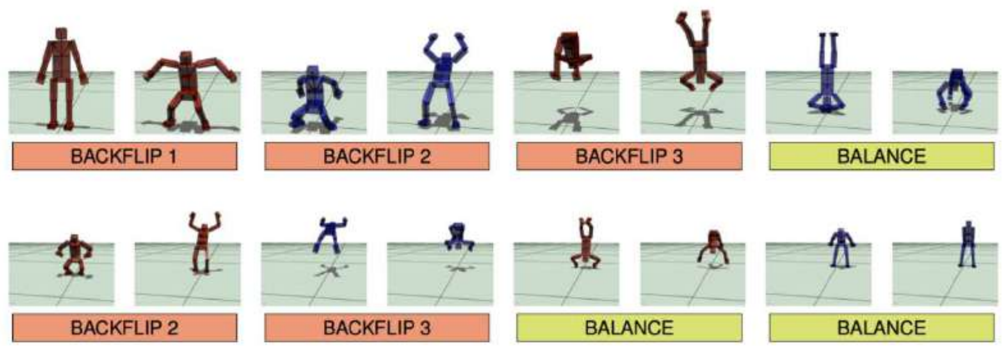
Trajectory Optimization synthesizes dynamic motions for complex robotic systems



[DARPA Robotics Challenge 2015]

[Foehn RSS 2017]



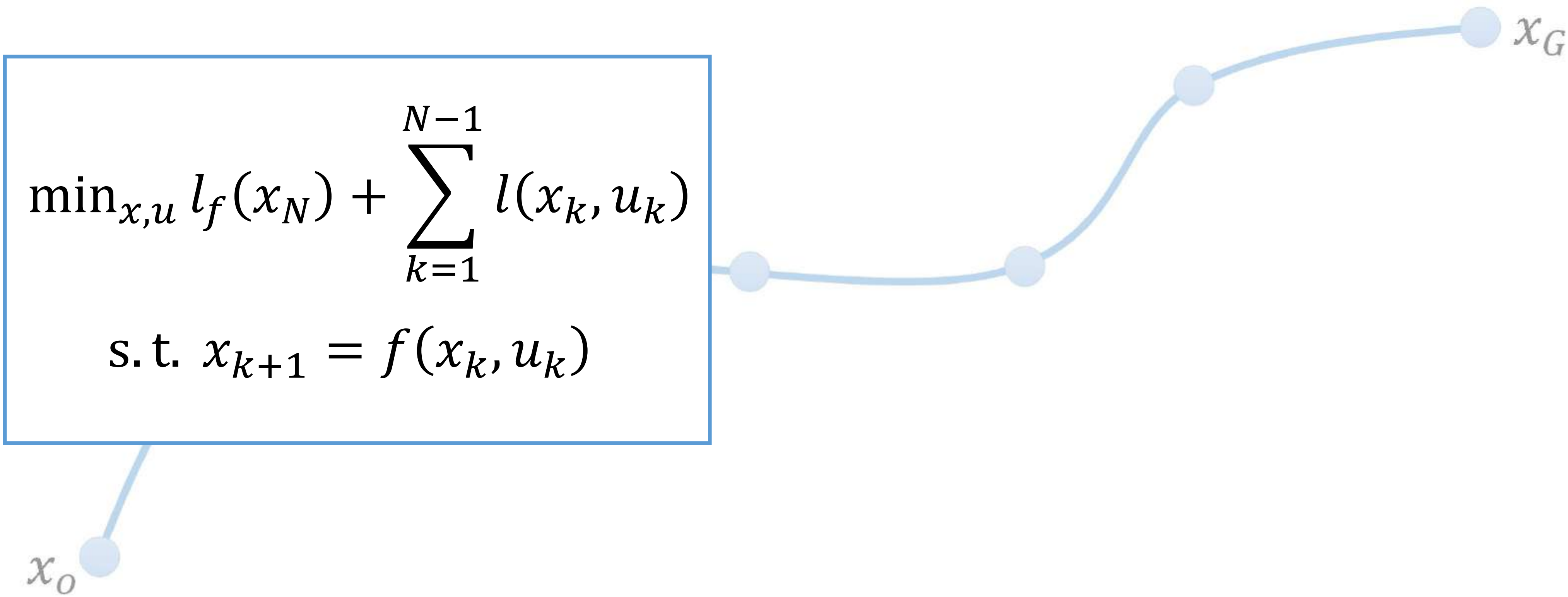


[Al Borno TVCG 2013]

Trajectory optimization minimizes a discrete time cost function subject to dynamics constraints

$$\min_{x,u} l_f(x_N) + \sum_{k=1}^{N-1} l(x_k, u_k)$$

$$\text{s.t. } x_{k+1} = f(x_k, u_k)$$



Dynamic Programming solves this problem through the recursive Bellman equation

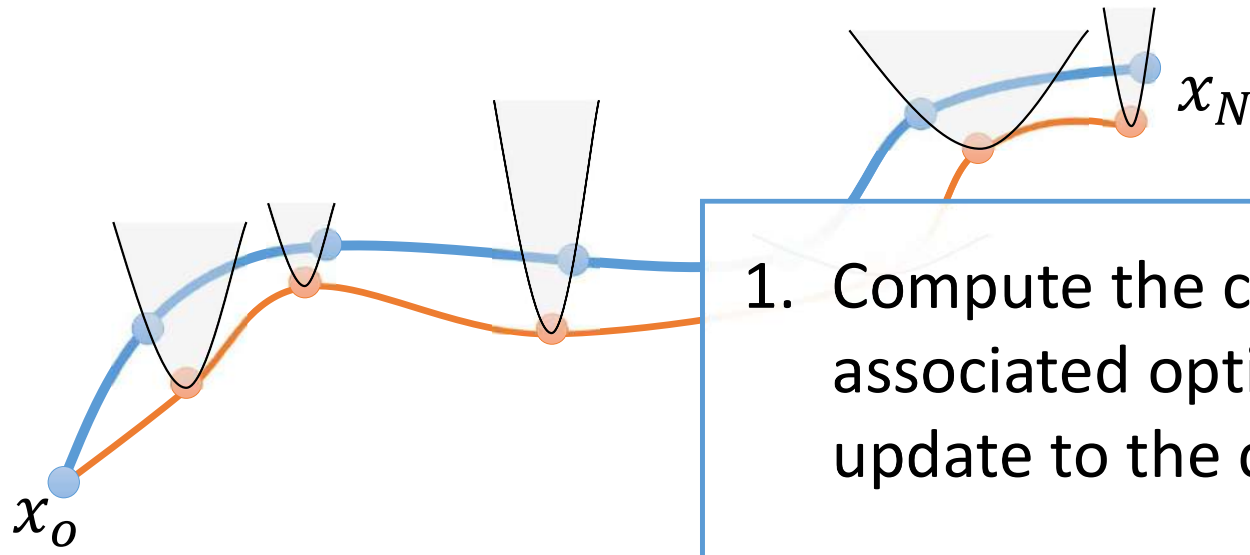
$$\min_{x,u} l_f(x_N) + \sum_{k=1}^{N-1} l(x_k, u_k)$$

$$\text{s.t. } x_{k+1} = f(x_k, u_k)$$

$$V_k(x) = \min_u l(x,u) + V_{k+1}\big(f(x,u)\big)$$

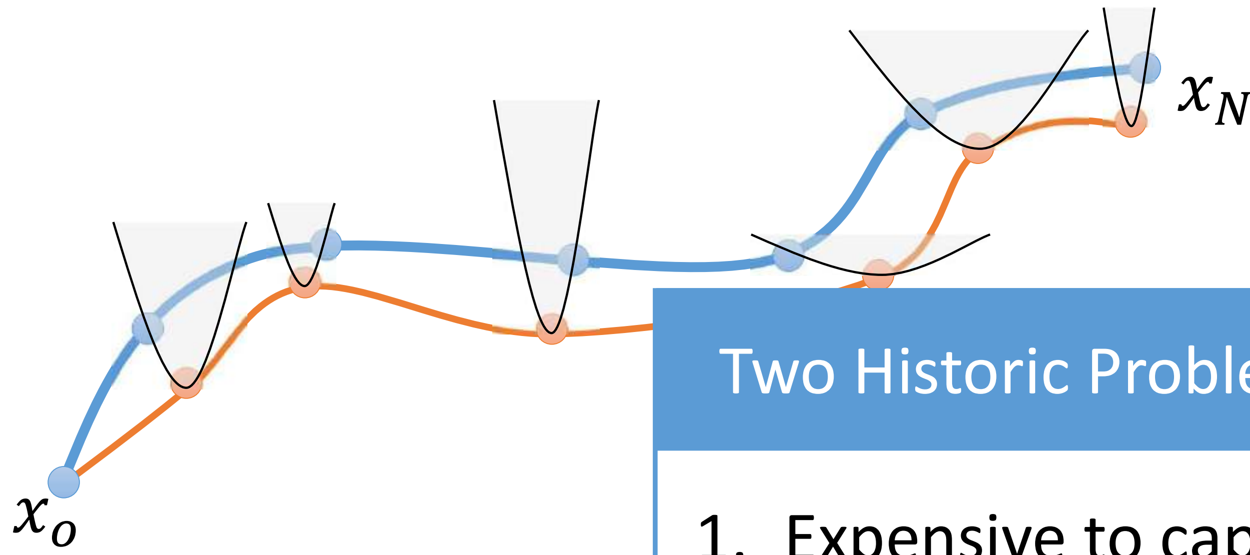
$$V_N(x_N) = l_f(x_N)$$

DP methods (DDP/SLQ/iLQR) use quadratic approximations around a nominal trajectory



1. Compute the cost-to-go and the associated optimal feedback control update to the controls **backward** in time
2. Simulate the system **forward** in time to create a new nominal trajectory
3. Repeat this process until convergence

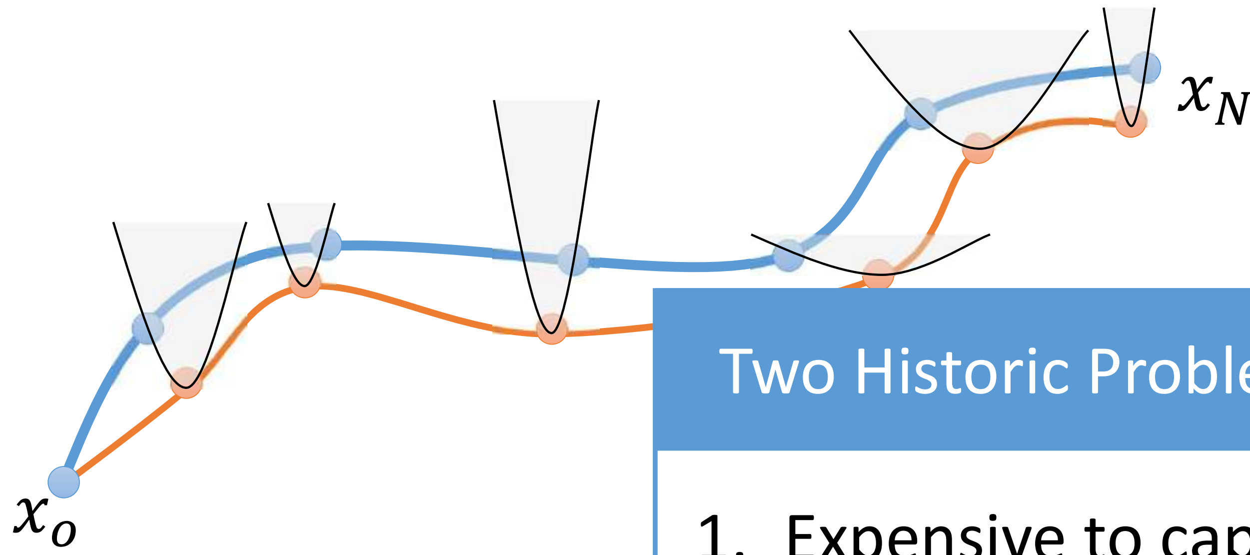
DP methods (DDP/SLQ/iLQR) use quadratic approximations around a nominal trajectory



Two Historic Problems with DP Algorithms

1. Expensive to capture the 2nd order information
2. Hard to enforce constraints

DP methods (DDP/SLQ/iLQR) use quadratic approximations around a nominal trajectory



Two Historic Problems with DP Algorithms

1. Expensive to capture the 2nd order information

2. **Hard to enforce constraints**

Recent research into adding constraints to DP like algorithms has taken two general paths

QP Methods

$$\min_{x,u}\ l_f(x_N) + \sum_{k=1}^{N-1} l(x_k, u_k) \quad \text{s.t. } x_{k+1} = f(x_k, u_k) \quad b_l \leq u \leq b_u$$

[Tassa ICRA 2014]
[Xie ICRA 2017]
[Farshidian ICRA 2017]

Penalty Methods

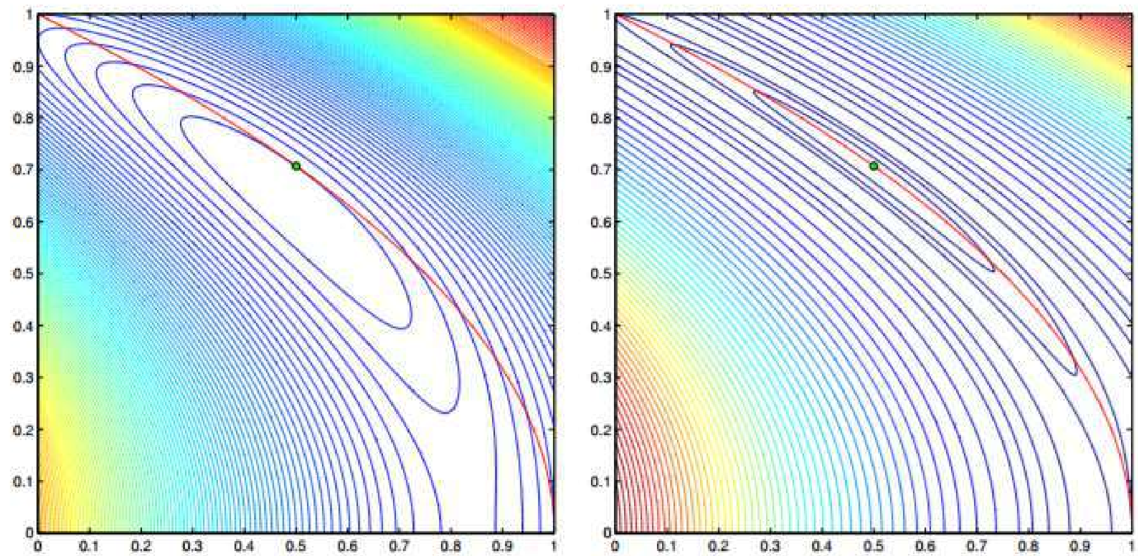
$$\min_{x,u}\ l_f(x_N) + \sum_{k=1}^{N-1} l(x_k, u_k) + \mu\|\phi(x_k, u_k)\|^2 \ \text{ s.t. } x_{k+1} = f(x_k, u_k)$$

[van den Berg ACC 2014]
[Farshidian ICRA 2017]
[Neunert RAL 2017]

Quadratic penalty methods are popular but can lead to numerical ill conditioning

Penalty Methods

$$\min_{x,u}\ l_f(x_N) + \sum_{k=1}^{N-1} l(x_k, u_k) + \mu\|\phi(x_k, u_k)\|^2 \quad \text{s.t. } x_{k+1} = f(x_k, u_k)$$

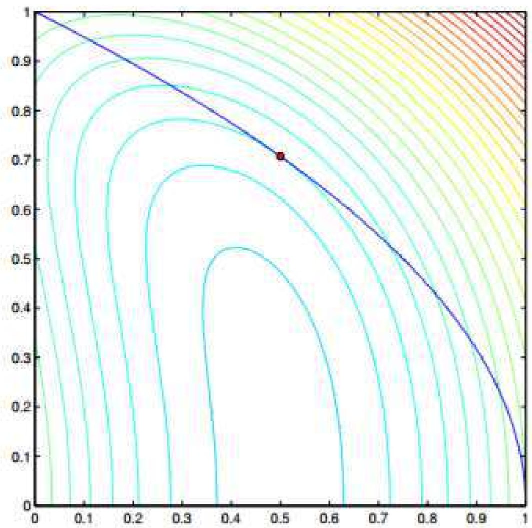


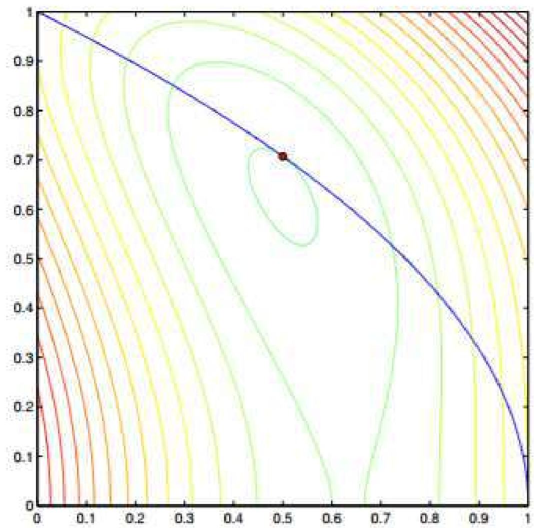
[Gould 2006]

Augmented Lagrangian methods show promise for trajectory optimization problems

Augmented Lagrangian Methods

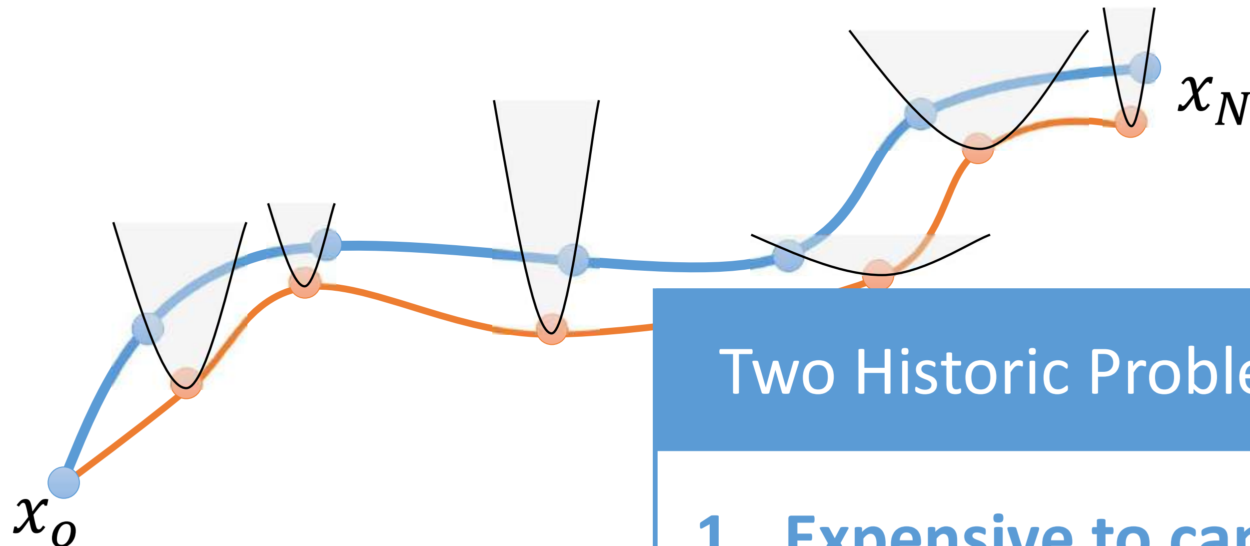
$$\min_{x,u}\ l_f(x_N) + \sum_{k=1}^{N-1} l(x_k, u_k) + \mu\|\phi(x_k, u_k)\|^2 + \lambda^T g(x_k, u_k) \quad \text{s.t. } x_{k+1} = f(x_k, u_k)$$





[Gould 2006]

DP methods (DDP/SLQ/iLQR) use quadratic approximations around a nominal trajectory

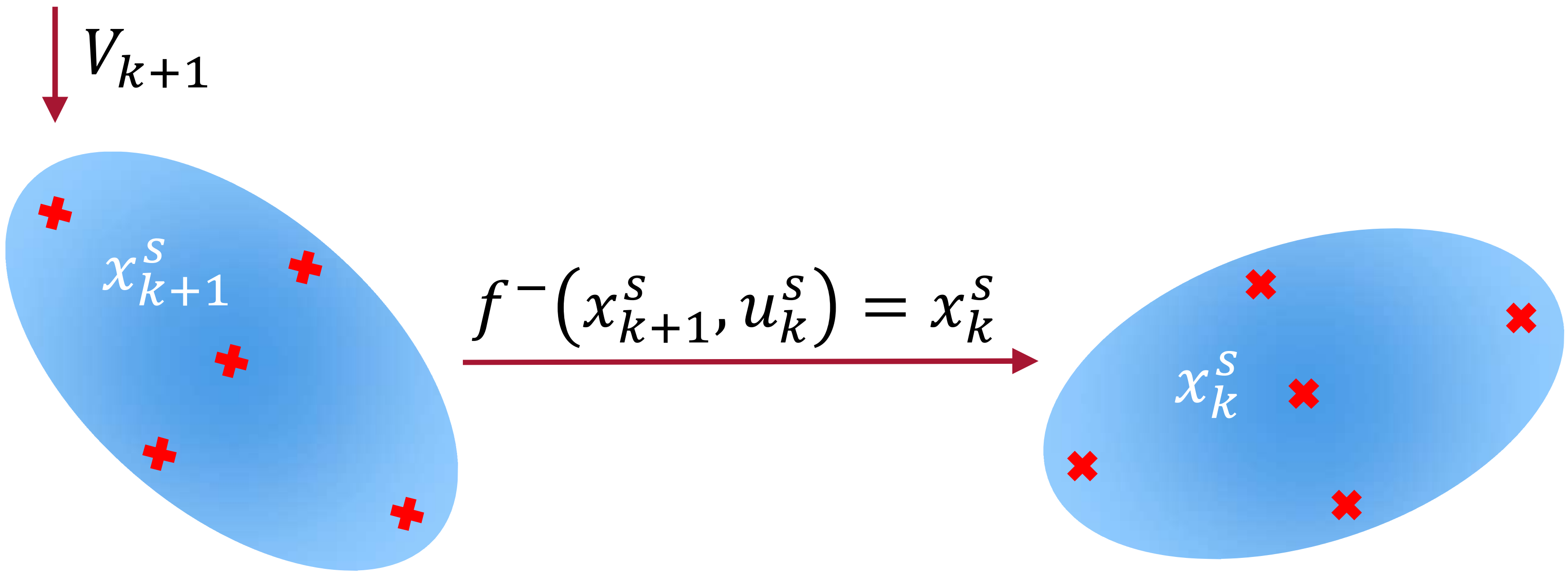


Two Historic Problems with DP Algorithms

1. **Expensive to capture the 2nd order information**

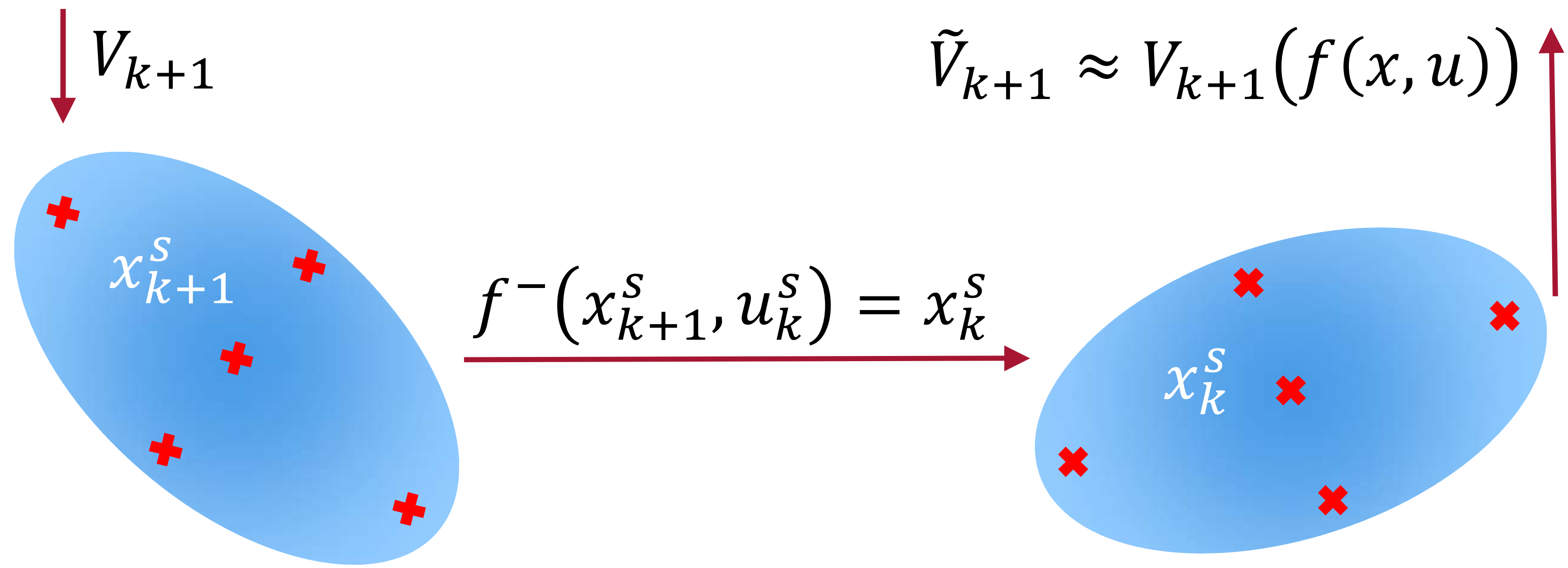
2. Hard to enforce constraints

UDP takes inspiration from the Unscented Kalman Filter to approximate the Hessian



[CDC 2016]

UDP takes inspiration from the Unscented Kalman Filter to approximate the Hessian

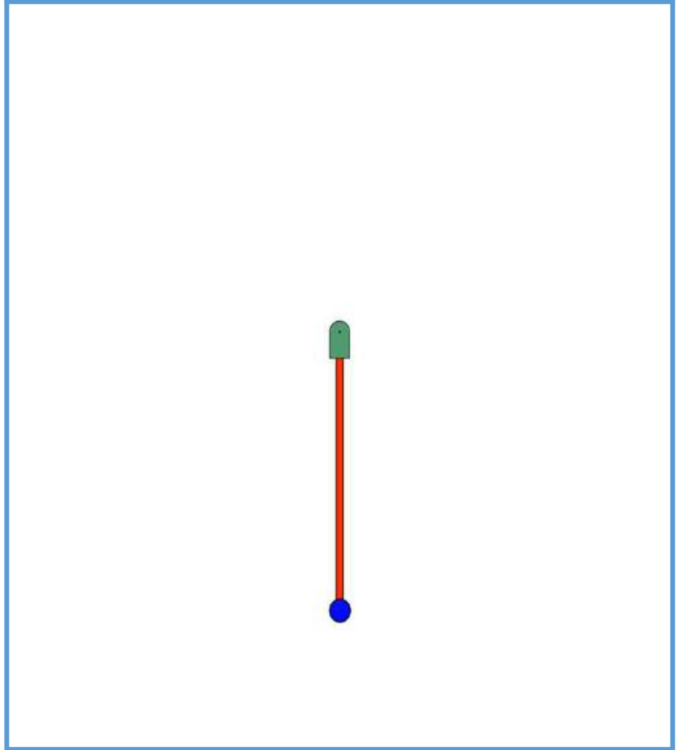


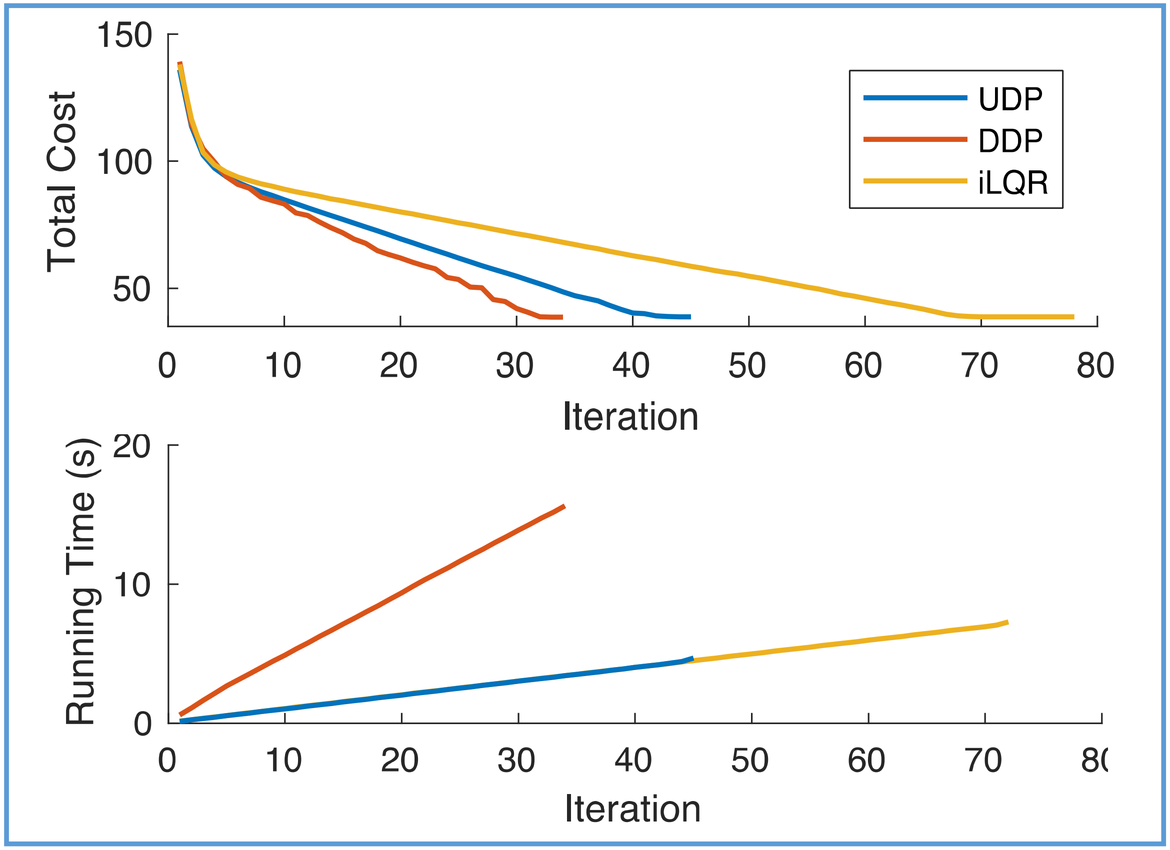
$$V_k(x) \approx \min_u l(x,u) + \tilde{V}_{k+1}$$

[CDC 2016]

Experimentally UDP captures 2nd order information with first order per-iteration cost

Pendulum Swing Up Task





[CDC 2016]

Constrained Unscented Dynamic Programming (CUDP)

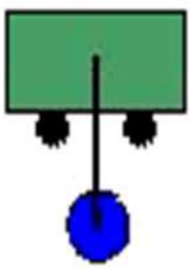
1. Compute the cost-to-go **(including constraint costs)** and the associated optimal feedback control update to the controls **backward** in time **using the unscented transform**

2. Simulate the system **forward** in time to create a new nominal trajectory

3. Repeat this process until convergence

4. **At convergence test for constraint satisfaction and if not update μ, λ and go back to step 1**

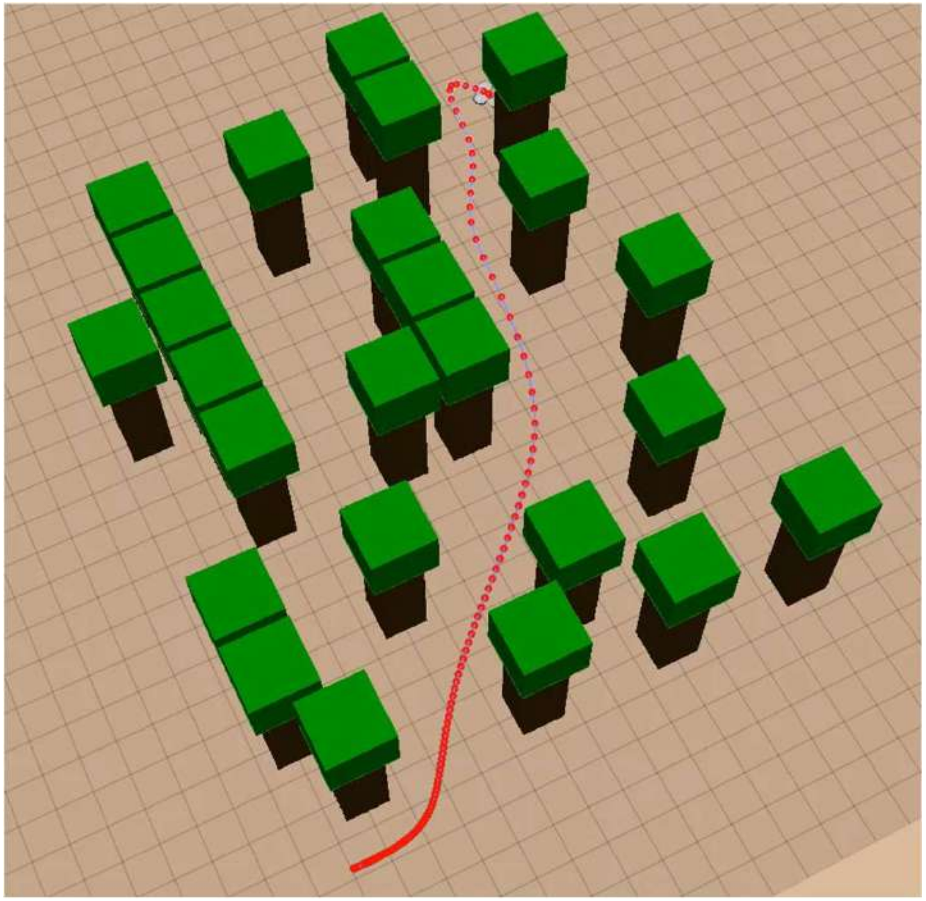
Precise constraint satisfaction requires both the unscented transform and augmented Lagrangian



	Φ < 1e-2	Φ < 1e-4	Φ < 5e-7
Penalty iLQR	✓	✗	✗
Penalty UDP	✓	✗	✗
AL iLQR	✓	✓	✗
AL UDP (CUDP)	✓	✓	✓

Constraints	• Torque Limit on motor • Final state position and velocity constraint

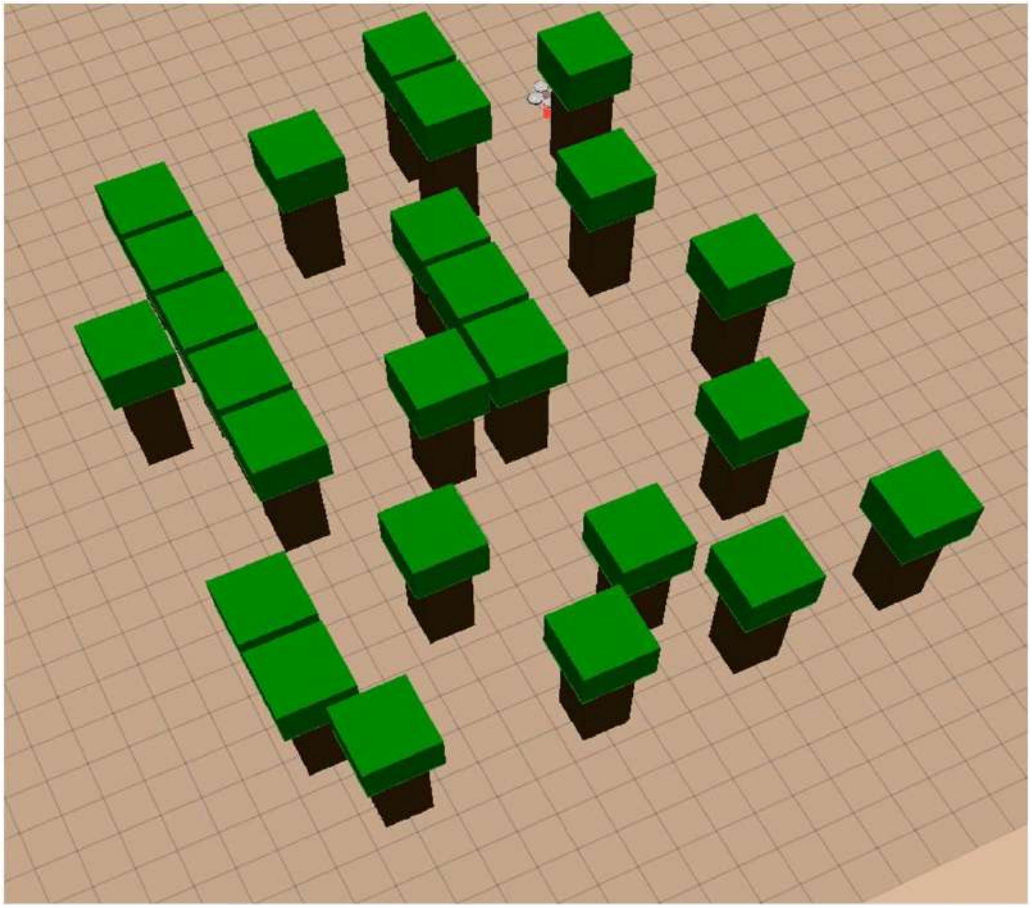
Precise constraint satisfaction requires both the unscented transform and augmented Lagrangian



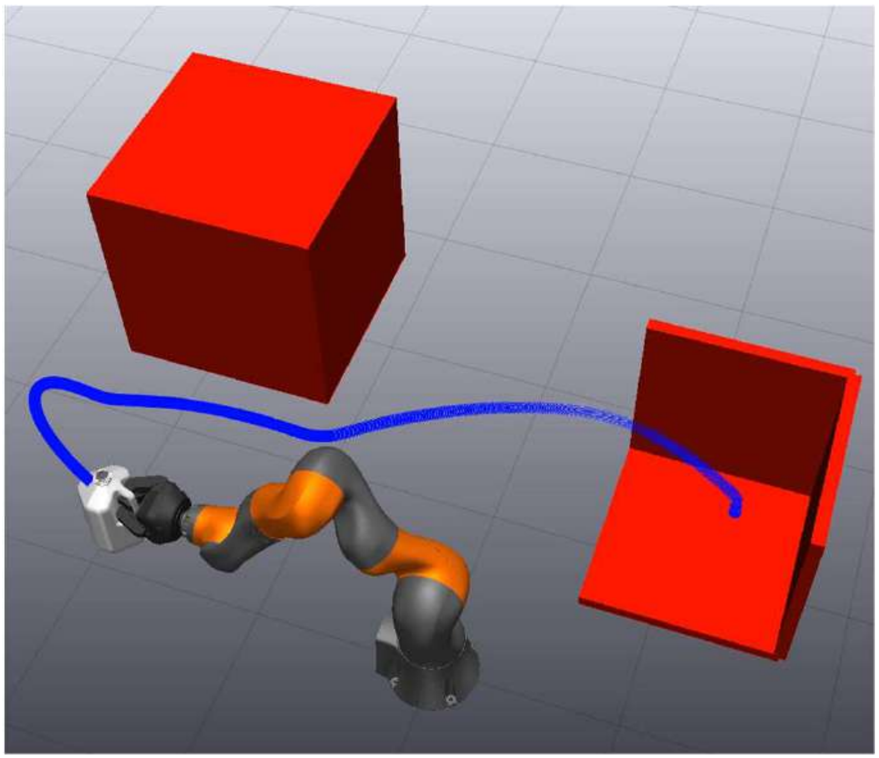
	Φ < 1e-2	Φ < 1e-4	Φ < 1e-6
Penalty iLQR	✓	✗	✗
Penalty UDP	✓	✓	✗
AL iLQR	✓	✗	✗
AL UDP (CUDP)	✓	✓	✓

Constraints	• Torque Limits on motors • No-contact constraints with trees • Final state position and velocity constraint

CUDP can pass through constraint boundaries during early major iterations



Precise constraint satisfaction requires both the unscented transform and augmented Lagrangian



	5e-1 Precision	1e-2 Precision	5e-3 Precision
Penalty iLQR	✓	✗	✗
Penalty UDP	✓	✗	✗
AL iLQR	✓	✓	✗
AL UDP (CUDP)	✓	✓	✓

Constraints	• Torque Limits on motors • No-contact constraints with block and shelf • Final state position and velocity constraint

Constrained Unscented Dynamic Programming

- A **derivative-free** DDP/iLQR algorithm inspired by the Unscented Kalman Filter
- Uses **augmented Lagrangian** to handle nonlinear state and input constraints
- Provides **faster convergence** and **higher constraint precision** vs iLQR and penalty methods

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